Investigation #2
Exploring Geometric Properties Through Problems

1) Read through these five problems and find one that jumps out at you.
Each one focuses on a different kind of geometrical property.
2) Work on that problem!
3) Write up your thinking and your results, using prose, symbols, arguments, diagrams, and created objects.
4) Brainstorm a variation, extension, or generalization of your problem that you think is interesting. Record it. Investigate it.
5) Share your results with your classmates in a short presentation.

By the time you present your findings, you will be the local experts on your problems. You will have thought more about them than anyone else here, and that's including me. Be prepared to turn in your write-up and to share your results on **Thursday, September 22**.

Position

How many non-overlapping circles that have a diameter of one inch can you fit onto an ordinary 8.5 x 11 sheet of paper?

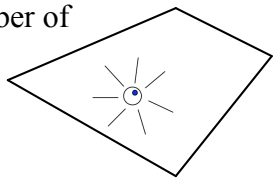
Size

Consider the attached grid as a map of city blocks with six marked intersections, where distance is measured by the number of blocks you have to walk to get from point to another.

- What is the shortest grid network that connects all six points? (That is, what is the shortest total distance of streets that would have to remain open so that travel among the six intersections would still be possible even if all the other streets were closed down?)
- Which point in the whole grid has the minimum total distance to the six points? (That is, where is the sum of the distances from that point to each of the indicated points the smallest?)

Shape

Imagine you needed to set up a security system at an art gallery, based on its 2D floor plan. You'd want to make sure that the cameras you installed would be able to see every part of the gallery, but you wouldn't want to buy more cameras than you had to. Assume that the cameras have a 360º turning radius and can see things as far away as you please, but that they can't see through walls. What can you say about how the shape of your gallery affects the number of cameras you'll need?



Dimension

The plane is two-dimensional and as such can be given a coordinate system with two variables—think of the standard x-y system. The same thing can be done with the surface of the earth, and this is usually accomplished using latitude and longitude. Create and describe coordinate systems for each of the following surfaces: a cube, a torus, and a barbell (see attached pictures). Giving multiple possibilities for each would be great, as would designing alternate coordinate systems for the plane and the sphere.

Connection

Consider the attached triangular figure. Its three corners can be colored red, blue, and green—one each. Find a coloring of the remaining nodes so that

- the edge between the red and green corners is colored using only red and green, and so forth for the other two edges.
- the coloring contains no small triangle that has corners of all three colors.